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Electric Dipole Moments as Probes of CPT Invariance

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Electric dipole moments (EDMs) of elementary particles and atoms probe violations of T and P symmetries and consequently of CP if CPT is an exact symmetry. We point out that EDMs can also serve as sensitive probes of CPT-odd, CP-even interactions, that are not constrained by any other existing experiments. Analyzing models with spontaneously broken Lorentz invariance, we calculate EDMs in terms of the leading CPT-odd operators to show that experimental sensitivity probes the scale of CPT breaking as high as 10^{12} GeV.

Tests of fundamental symmetries play an important role in discerning the properties of nature at ultra-short distance scales. Initially suggested as an accurate test of parity conservation in strong interactions [1], the electric dipole moments (EDMs) of neutrons and heavy atoms provide an important test of P and T symmetries [2, 3, 4]. A non-relativistic Hamiltonian for a neutral particle of spin S can be written as the combination of two terms,

$$H = -\mu \mathbf{B} \cdot \frac{\mathbf{S}}{S} - d\mathbf{E} \cdot \frac{\mathbf{S}}{S}\ . \tag{1}$$

Under the reflection of spatial coordinates, $P(\mathbf{B} \cdot \mathbf{S}) = \mathbf{B} \cdot \mathbf{S}$, whereas $P(\mathbf{E} \cdot \mathbf{S}) = -\mathbf{E} \cdot \mathbf{S}$. Under time reflection, $T(\mathbf{B} \cdot \mathbf{S}) = \mathbf{B} \cdot \mathbf{S}$ and $T(\mathbf{E} \cdot \mathbf{S}) = -\mathbf{E} \cdot \mathbf{S}$. Thus, the presence of a non-zero d would therefore signify the existence of both parity and time-reversal violation. In a world with perfect CPT symmetry, a search for d would also be a direct test of CP symmetry. An assumption of CPT is well-justified in the field theory framework, as it rests on locality, spin-statistics and Lorentz invariance. Nevertheless, independent tests of CPT are warranted, and a number of searches in the K and B meson systems [5], as well as with electrons, muons and antiprotons have been pursued over the years. In this paper, we show that EDMs can serve as a sensitive probe of CPT violation, independent from other available tests. More specifically, we relax the assumption of Lorentz invariance thus enabling the breaking of CPT and study the EDMs induced by CPT-odd but CP-even interactions.

Suppose that the breaking of CPT symmetry comes from some unknown, presumably short-distance scale, physics and manifests itself in the interaction of Standard Model fields with external backgrounds that transform as vectors and tensors under the Lorentz group [6, 7]. The simplest possibility is to have a time-like condensation of a vector $n_\mu = (1, 0, 0, 0)$ that introduces a preferred frame. For simplicity we assume that n_μ coincides with the laboratory frame, but the results can be easily generalized for a generic frame. In the presence of such a vector, the EDM part of Hamiltonian (1) for the spin 1/2 particle can be rewritten as

$$\mathcal{L}_{\rm EDM} = \frac{-i}{2} d_{\rm CP} \overline{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi + d_{\rm CPT} \psi \gamma_\mu \gamma_5 \psi F_{\mu\nu} n^\nu \tag{2}$$

where $d_{\rm CP} + d_{\rm CPT} = d$. Thus, quite generically, the nil result for the neutron EDM searches provides a constraint on the combination $d_{\rm CP} + d_{\rm CPT}$. Introducing an axial four-vector of spin $\mathfrak{a}^\mu$ and four-velocity u^μ, we generalize (2) for a particle of arbitrary spin:

$$\mathcal{L}_{\rm EDM} = F_{\mu\nu} \mathfrak{a}^\nu (d_{\rm CP}\, u^\mu \ + \ d_{\rm CPT}\, n^\mu). \tag{3}$$

Allowing for more complicated backgrounds, we notice that the CPT-odd EDM-type correlation may also result from interaction with irreducible tensor $D^{\mu\nu\rho}$, symmetric in $\nu\rho$: $F_{\mu\nu} \mathfrak{a}^\rho D^{\mu\nu\rho}$. In the remainder of this paper, we analyze the structure of the CPT-odd and CP-even effective Lagrangian, deduce its consequences for the EDMs of neutrons and heavy atoms, and explore the posibility of distinguishing $d_{\rm CP}$ and $d_{\rm CPT}$ in experiment, should the non-zero EDMs be found.

CPT-odd, CP-even operators. In the framework where CPT violation is mediated by Lorentz violation, the CPT-odd interaction terms appear at odd dimensions [6]. All CPT-odd dimension three operators can be easily listed [6],

$$\mathcal{L}_3 = -\sum \bar{\psi}(a^\mu \gamma_\mu + b^\mu \gamma_\mu \gamma_5)\psi, \tag{4}$$

with a_μ and b_μ being Lorentz/CPT violating couplings with possible flavor dependence. Only certain types of CPT-violating dimension five operators were classified in the literature [8], and here we complement this list by including operators linear in the gauge field strength:

$$\mathcal{L}_5 = -\sum [c^\mu \overline{\psi} \gamma^\lambda F_{\lambda\mu} \psi \ + \ d^\mu \overline{\psi} \gamma^\lambda \gamma^5 F_{\lambda\mu} \psi \\ + \ f^\mu \overline{\psi} \gamma^\lambda \gamma^5 \widetilde{F}_{\lambda\mu} \psi \ + \ g^\mu \overline{\psi} \gamma^\lambda \widetilde{F}_{\lambda\mu} \psi]. \tag{5}$$

The sum spans different fermions of the SM and different gauge group, with $F_{\mu\nu}$ standing for the corresponding

Coefficient	Operator	C	P	T
a^0	$\overline{\psi}\gamma_0\psi$	$-$	$+$	$+$
b^0	$\overline{\psi}\gamma_0\gamma_5\psi$	$+$	$-$	$+$
c^0	$F_{\lambda 0}\overline{\psi}\gamma^\lambda\psi$	$+$	$+$	$-$
d^0	$F_{\lambda 0}\overline{\psi}\gamma^\lambda\gamma^5\psi$	$-$	$-$	$-$
f^0	$\widetilde{F}_{\lambda 0}\overline{\psi}\gamma^\lambda\gamma^5\psi$	$-$	$+$	$+$
g^0	$\widetilde{F}_{\lambda 0}\overline{\psi}\gamma^\lambda\psi$	$+$	$-$	$+$

TABLE I: C, P, T properties of dimension three and five Lorentz violating CPT-odd operators. Only one operator proportional to d^0 is both P and T odd and capable of inducing EDMs.

field strength. Assuming that the vector backgrounds are time-like and invariant under C, P and T reflections, we classify the properties of operators (4) and (5) under these discrete symmetries in Table 1. There is only one operator that is odd under parity and time reversal, and thus our further analysis concentrates only on d^μ.

It is convenient to classify these operators at the scale of 1 GeV, where only light quark fields, gluons, photons, electrons and muons are the remaining degrees of freedom, while weak bosons and heavy quarks are already decoupled. Taking a quark field ψ_q with the electric charge Q_q, and using the full equation of motion in the electromagnetic and strong backgrounds,

$$iD_\mu\gamma^\mu\psi_q \equiv (i\partial_\mu - g_s t^a A^a_\mu - eQ_qA_\mu)\gamma^\mu\psi_q = m_q\psi_q, \quad (6)$$

we deduce an identity that relates gluon and photon-containing operators for quarks:

$$\begin{aligned}\bar\psi_q(eQ_qF_{\mu\nu} + g_st^aG^a_{\mu\nu})\gamma^\nu\gamma_5\psi_q &= -i\bar\psi_q[D_\mu, D_\nu\gamma^\nu\gamma_5]\psi_q \\ = 2m_i\bar\psi_qD_\mu\gamma_5\psi_q &= m_q\bar\psi_q[D_\nu\gamma^\nu, \gamma_\mu\gamma_5]\psi_q = 0. \quad (7)\end{aligned}$$

Here $[,]$ is the commutator. Eq. (7) effectively reduces the number of independent quark operators, and we choose to eliminate $\bar\psi_q g_s t^a G^a_{\mu\nu}\gamma^\nu\gamma_5\psi_q$ by expressing it via $\psi_q eQ_qF_{\mu\nu}\gamma^\nu\gamma_5\psi_q$. Remarkably, there is no CPT-odd, CP-even operators for Dirac particles that have only electromagnetic interactions, such as muons and electrons, because in this case Eq. (7) degenerates to an identity $\psi_eF_{\mu\nu}\gamma^\nu\gamma_5\psi_e = 0$. It turns out that the vanishing of this efective operator is well known in the standard CP-odd EDM computations. The correction to the electron Hamiltonian created by operator $\psi_eF_{0\nu}\gamma^\nu\gamma_5\psi_e$ is proportional to the product of electric field and relativistic spin operator $\mathbf{\Sigma}$, $\mathbf{E\Sigma}$. This product can be represented as a result of the commutator of another operator with the full Dirac Hamiltonian, $\mathbf{E\Sigma}= (1/e)[\mathbf{\Sigma}\nabla, H]$. Therefore, the expectation value of $\mathbf{E\Sigma}$ over any eigenstate of H is zero [3, 9], which is another way of stating that $\psi_eF_{0\nu}\gamma^\nu\gamma_5\psi_e$ vanishes on shell.

Taking these identities into account, we write down the effective T, P, CPT-odd Lagrangian at 1 GeV scale in a remarkably simple form, that contains only three terms:

$$\mathcal{L}_{\rm CPT} = \sum_{i=u,d,s} d^\mu_i\bar q_i\gamma^\lambda\gamma^5F_{\lambda\mu}q_i. \quad (8)$$

This is a rather compact form compared to a usual CP-odd effective Lagrangian where a few dozens of terms have to be taken into account [4].

An important difference between CP-odd and CPT-odd EDMs comes from the $SU(2)\times U(1)$ properties of Eq. (8). CP-odd effects require helicity flip and thus correspond to dimension 6 operators above the electroweak scale, decoupling as $1/\Lambda^2_{\rm CP}$ as the scale of CP violation $\Lambda_{\rm CP}$ gets larger. One can easily see that CPT-odd terms (8) correspond to genuine dimension 5 operators such as $\bar q_{R(L)}\gamma^\lambda\gamma^5F_{\lambda\mu}q_{R(L)}$ and $\bar q_L\gamma^\lambda\gamma^5\tau^aF^a_{\lambda\mu}q_L$ and do not require chirality flip. Consequently, CPT-odd physics decouples only linearly, $d_{\rm CPT}\propto\Lambda^{-1}_{\rm CPT}$. Combination of present day limit on neutron EDM with the linear decoupling property furnishes the sensitivity to the scales of CPT violation as large as

$$\Lambda_{\rm CPT}\sim(10^{11}-10^{12})\ {\rm GeV}. \quad (9)$$

Future generation experiments could potentially probe CPT-violating physics all the way to the Planck scale, being limited only by the prediction of the Kobayashi-Maskawa (KM) model for the neutron EDM at the level of $10^{-31}-10^{-33}e$cm.

Signatures of CPT-odd EDMs. There are three main groups of observable EDMs, which include EDMs of neutrons, diamagnetic atoms (Hg, Xe, etc.) and paramagnetic atoms (Tl, Cs, etc.). A rather simple structure of the CPT-odd efective Lagrangian helps to determine the dependence of these observables on different d^μ_i in (8).

The QCD calculations of conventional CP-odd EDMs [4] are very close to a constituent quark model prediction, $d_n\simeq\frac{4}{3}d_d-\frac{1}{3}d_u$, with the contribution of the s-quark being zero. In the CPT-odd case, we use matrix elements of the axial-vector charges of light quarks inside a nucleon, which can be obtained from the nucleon spin structure functions [10]. This way, to $\sim$20% accuracy, we get

$$d_n\simeq 0.8d^0_d-0.4d^0_u-0.1d^0_s. \quad (10)$$

Using $|d_n| < 3\times10^{-26}e$cm [2] and barring significant cancellation between the constituents, we conclude that CPT-odd EDMs of light quarks are limited at $O(10^{-25}e{\rm cm})$.

The measurements of EDMs of diamagnetic atoms are usually quite competitive with d_n due to color EDM contributions to the CP-odd pion-nucleon coupling constant $\bar g_{\pi NN}$ [3, 4]. As we already noted, interactions (8) preserve quark chirality, and involve a photon field, thus leading to a strong suppression of $\bar g_{\pi NN}(d^\mu_q)$, which makes the T-odd pion exchange ineffective. Consequently, the EDM of the diamagnetic atoms are induced by the EDMs of the valence nucleons. For the most important case of mercury EDM [11], we have

$$d_{\rm Hg} \simeq -5\times 10^{-4}(d_n + 0.1 d_p) \\ \simeq -5\times 10^{-4}(0.74 d_d^0 - 0.32 d_u^0 - 0.11 d_s^0), \qquad (11)$$

and an approximate relation $d_{\rm Hg}/d_n \sim -5\times 10^{-4}$ could be interpreted as a signal consistent with CPT violation should the nonzero $d_{\rm Hg}$ and d_n be found. Due to absence of CPT-odd electron EDM operator, EDMs of paramagnetic atoms are predicted to be extremely suppressed.

An unambigous separation of CP-odd and CPT-odd EDM terms in (3) may come from measuring the difference of their relativistic effects. The CP-odd EDM interacts with the magnetic field and leads to the precession of the spin relative to $[\mathbf{B}\times\mathbf{v}]$, while the CPT-odd component does not contribute to the precession for a particle on a circular orbit. Thus, the experimental proposal of measuring deuteron EDM in the storage ring [12] would in principle have capabilities of separating the two effects, as perpendicular $\mathbf{B}$ and $\mathbf{E}$ would be employed in the experimental set-up. In practice, the signal of spin precession due to the CPT-odd EDM is not exactly zero but suppressed by the deuteron anomaly, $|a_D| = 0.143$, because of the $|\mathbf{E}| = |a_D\mathbf{B}|$ choice [12]. The suppression of the deuteron $d_{\rm CPT}$ signal measured in the storage ring relative to d_n is opposite to the case of $d_{\rm CP}$ where an enhancement of $d_D/d_n \sim 5$ is expected [13] due to the CP-odd pion exchange.

Naturallness. Since there are many other observables sensitive to Lorentz/CPT violation given by dimension three operators (4), it is important to investigate whether operators of dimension five (5) may influence these observables through quantum loops. It is easy to see, for example, that the last dimension five operator in (5), $g^\mu \widetilde{F}_{\mu\nu}\overline{\psi}\gamma_\nu\psi$, produces a quadratically divergent result for dimension three term, $b^\mu\psi\gamma_\mu\gamma_5\psi$, already at one loop. Even with a modest choice of the cutoff, the contribution to b_μ will significantly exceed present experimental bounds of order 10^{-31} GeV, modulo an extreme fine-tuning. It turns out that EDM operators d^μ are protected against transmutation to a_μ and b^μ to a high loop order because of their difference in CP. Thus, only loops with intrinsic CP violation can convert d^μ into a^μ or b^μ. In the SM this is rather difficult to achieve, as the violation of CP symmetry in the flavour-conserving channel happens minimum at three loops, and is further suppressed by the Kobayashi-Maskawa mixing angles and quark Yukawa couplings. A crude estimate of dimension three operators resulting from multi-loop CP-violating corrections gives an addmittedly imprecise prediction for a light quark,

$$a^\mu, b^\mu \sim d^\mu(10^{-20} - 10^{-18})\times {\rm GeV}^2. \qquad (12)$$

This provides sensitivity to d_μ up to 10^{-12} GeV^{-1}, which is essentially the same sensitivity as (9). Therefore a detectable signal from the CPT-odd EDMs induced by a vector background would likely come accompanied by b^μ, which could be searched for via *e.g.* sidereal modulation of spin precession frequencies [6]. A difference of down and strange a_μ terms can be searched for with the neutral K mesons producing a typical bound on $|a_s^0 - a_d^0|$ of order $\sim 10^{-19} - 10^{-20}$ GeV. Through the loop effects, this amounts to sensitivity to d_q^0 terms on the order of 10^{-5} GeV^{-1}, which is significantly less sensitive than (9).

Tensor backgrounds. What if the nature of CPT-violation is so intricate as to give rise to an external rank-three tensorial background $D^{\mu\nu\rho}$? In this case the the T, P and CPT odd interaction $F_{\mu\nu}\mathfrak{a}^\rho D^{\mu\nu\rho}$ induces the EDM-like signatures via an anisotropic effective Hamiltonian for the spin:

$$H = -\mu\mathbf{B}\cdot\frac{\mathbf{S}}{S} - \mathcal{D}^{ij}E_i\cdot\frac{S_j}{S}. \qquad (13)$$

Here $\mathcal{D}^{ij}$ is the traceless symmetric tensor with spatial components, $\mathcal{D}^{ik} = \mathcal{D}^{i[0k]} + \mathcal{D}^{k[0i]}$. The tensor interaction in (13) creates a correction to the spin precession frequency proportional to $E_iB_k\mathcal{D}^{ik}$ which changes sign under the reversal of the electric field. The effect averages to zero if the orientation of parallel $\mathbf{E}$ and $\mathbf{B}$ fields is randomly changing relative to the external tensor $\mathcal{D}^{ik}$ due to its tracelessness. However, in EDM experiments such averaging is not done. Therefore, $E_iB_k\mathcal{D}^{ik}$ gives an EDM signature, which in addition changes during the day because of the change of the orientation of a laboratory relative to $\mathcal{D}^{ik}$ if, of course, the frame that breaks Lorentz invariance is not related to the Earth itself. Generically, one expects 12 and 24 hour modulations of the EDM signal due to the CPT-odd tensor background. The structure of operators leading to (13) is more complex than in the vector case. In particular, the electron operator, $\bar{e}F_{\mu\nu}\gamma_\rho\gamma_5 e D^{\mu\nu\rho}$ does not vanish, and leads to the EDMs of a paramagnetic atom, albeit with the matrix element suppressed by a factor of ~ 10 relative to the CP-odd case. As in the vector case, the EDMs of diamagnetic atoms are induced by the EDMs of valence nucleons. Finally, tensor backgrounds are protected against transmutation to lower dimensional operators.

In conclusion, we point out that EDMs put stringent limits on a new type of CPT-odd CP-even interactions that is not constrained by other tests of Lorentz invariance and CPT. The scale of CPT-breaking probed by current versions of EDM experiments is as high as 10^{12} GeV. The unambigous separation of CPT-odd and CP-odd effects would require EDM experiments with antiparticles, which might be a formidable challenge. Instead, we point out the main pattern in EDM observables consitent with CPT violation: nuclear and atomic EDMs will be induced by the EDMs of neutrons and protons, while electron EDM and T-odd nuclear forces are largely ineffective in the CPT-odd case.

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